Evolution of Massive Blackhole Triples I — Equal-mass binary-single systems

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ABSTRACT

We present the result of N-body simulations of dynamical evolution of triple massive blackhole (BH) systems in galactic nuclei. We found that in most cases two of the three BHs merge through gravitational wave (GW) radiation in the timescale much shorter than the Hubble time, before ejecting one BH through a slingshot. In order for a binary BH to merge before ejecting out the third one, it has to become highly eccentric since the gravitational wave timescale would be much longer than the Hubble time unless the eccentricity is very high. We found that two mechanisms drive the increase of the eccentricity of the binary. One is the strong binary-single BH interaction resulting in the thermalization of the eccentricity. The second is the Kozai mechanism which drives the cyclic change of the inclination and eccentricity of the inner binary of a stable hierarchical triple system. Our result implies that many of supermassive blackholes are binaries.

Subject headings: black hole: physics — black hole: binary — galaxies: nuclei — stellar dynamics — Three-body problem:general — Gravitational Wave Radiation : LISA — methods: n-body simulations

1. Introduction

When two galaxies merge, the central BHs sink toward the center of the merger remnant and form a binary system (Begelman, Blandford, & Rees 1980, henceforth BBR). Whether or not this binary BH can merge in the timescale shorter than the Hubble time is an important question for the understanding of the evolution and growth of BHs and galactic nuclei.

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BBR predicted that the hardening of the BH binary through the stellar dynamical interaction with field stars (FSs) would slow down after the loss cone is depleted. After the depletion, the growth will be driven by the refilling of the loss cone by two-body relaxation, and a simple theoretical estimate results in the merger timescale orders of magnitude larger than the Hubble time.

Whether or not this slowing down of evolution actually occur has been studied with N-body simulations (Quinlan & Hernquist 1997; Milosavljević & Merritt 2003), Until very recently, however, there has been serious discrepancy between theory and simulation results, and also among simulation results. Theory predicts that the timescale must be proportional to the number of particles used to express the parent galaxy, since the relaxation timescale is (neglecting the weak $\log N$ term) proportional to the number of particles in the system N. The results of N-body simulations range from no dependence on N (Chatterjee et al. 2003; Milosavljević & Merritt 2001) to $N^{1/3}$ (Makino 1997).

Makino & Funato (2004) performed N-body simulations with much larger number of particles (up to 10^6) and for longer time compared to previous works, and concluded that the N-dependence of the evolution timescale of BH binary is consistent with the theoretical prediction. Berczik, Merritt, & Spurzem (2005) obtained similar result, by performing even longer simulations for somewhat smaller N (up to 256k).

Thus, as far as the stellar dynamical effects are concerned, BH binary, once depleted the loss cone, would not merge within the Hubble time, unless there remain lots of gas in the central region of merger remnants.

If BHs in galaxies, in particular giant ellipticals, are mostly binaries, growth of the BH through merging might be difficult. If one galaxy with binary BH and the other with single BH merge, the central BHs form a triple system, which is generally unstable. The most likely outcome would be the ejection of at least one BH, preventing the growth of the central BH binary. In fact, triple interaction of BHs was first considered as an mechanism to eject BHs from the center of a galaxy(Saslaw et al. 1974). The timescale for the merging through GW radiation is given by

$$t_{gr} = 2 \times 10^{15} \left(\frac{10^8 M_\odot}{m_B}\right)^3 \left(\frac{a}{1\mathrm{pc}}\right)^4 g(e), \tag{1}$$

$$g(e) = \frac{(1-e^2)^{7/2}}{1 + (73/24)e^2 + (37/96)e^4}, \tag{2}$$

where m_B is the mass of BHs (assuming equal masses), a and e are the semi-major axis and eccentricity of the binary. In the case of a BH with $10^8 M_\odot$, the orbital separation must

become less than 0.1 pc in order for the merging timescale to become shorter than the Hubble time, if we assume the average thermalized value of $e = 0.7$.

Since the gravitational wave timescale depends on the eccentricity very strongly, if some mechanism drives the binary BH to high eccentricity the merging timescale might become short. Makino & Ebisuzaki (1994) argued that the maximum eccentricity a binary BH would experience is much higher than the average value, since a binary BH must undergo around 10 strong single-binary scattering events before ejecting out a single BH. They concluded, using simple analytic arguments, that the binary BH in a triple BH system would merge, for the case of typical BH of $10^8 M_\odot$ in a typical elliptical galaxy within the Hubble time. Another possibility is the change of eccentricity through Kozai mechanism(Kozai 1962).

In this paper, we investigate the evolution of triple BH systems by means of direct N-body simulations. Our initial model consists of an N-body realization of a spherical galaxy and three BH particles. We started with hierarchical configuration in which two BH particles initially form a binary. The third BH particle is placed at some distance to the binary. We changed the initial orbit of the third particle, to see its effect on the evolution of the binary. We incorporated the effect of GW radiation on the relative orbits of BH particles.

We found that if the initial orbits of three BHs are coplanar and if the total angular momentum is small, the binary BH hardens through repeated binary-single BH interactions. The interaction stops either when one BH is ejected out of the galaxy or when the BH binary merges through GW radiation. Which of the two dominates depends on the depth of the potential well of the parent galaxy. Merging is more likely for deeper potential. For typical giant ellipticals, potential is deep enough that merging dominates.

When the orbits are not coplanar, hierarchical triples are formed and the inner binary evolves through Kozai mechanism, in a fair fraction of cases. Kozai mechanism leads the large-amplitude oscillation of eccentricity. Thus, merging is more likely in three-dimensional configuration than in coplanar systems.

The structure of this paper is as follows. In section 2, we describe the initial models and method of our numerical simulation. In section 3, we show the result of the cases in which three black holes are initially in coplanar orbits. In section 4, we show the result of more general cases of non-coplanar initial conditions. We summarize our result of numerical simulation in section 5.

2. Initial Models and Numerical Methods

2.1. Models

The initial setup is schematically shown in figure 1. We placed three BH particles in N-body models of a spherical galaxy. The quantitative properties of models and initial conditions of our simulation is summarized in table 1.

For the galaxy model, we used a King model with $W_0 = 7$, where W_0 is the nondimensional central potential of King models (King 1966; Binney & Tremaine 1987). We performed several runs changing the number of particles in a galaxy from 16K to 128K. We adopted the the standard N-body units (Heggie & Mathieu 1986), in which $M_{gal} = 1.0, E_{gal} = -0.25, G = 1$. Here, M_{gal} and E_{gal} are the total mass and total binding energy of the galaxy and G is the gravitational constant. The relation to the physical units will be discussed in section 2.5.

We placed three blackholes in the parent galaxy model. All blackhole particles have the same mass of 0.01 in the N-body units. Two blackholes are initially placed as a binary with eccentricity $e \sim 0.7$ in the center of the galaxy model. The orbital plane of this binary is the $x - y$ plane. the third blackhole is placed at position (1,0,0).

We have changed the initial velocity of the third particles in several different ways. In the simplest case, it is placed at rest. In one series, we gave initial velocity along y axis, and in another series along z and in the third one in $y - z$ plane. Tables 2 and 3 list all runs we performed. We scaled the positions and velocities of stars so that the total energy of the system, including that of black hole particles, becomes $-1/4$.

2.2. Equations of Motion

The equation of motion for field particles is given by

$$\frac{d^2\mathbf{r}_{f,i}}{dt^2} = \mathbf{F}_{ff,i} + \mathbf{F}_{fB,i}, \tag{3}$$

where $\mathbf{F}_{ff,i}$ and $\mathbf{F}_{fB,i}$ are acceleration due to field stars and BH particles respectively. They are given by

$$\mathbf{F}_{ff,i} = \sum_{j}^{N_f} \frac{-m_{f,j}\mathbf{r}_{ff,ij}}{\left(r_{ff,ij}^2 + \epsilon_{ff}^2\right)^{3/2}}, \tag{4}$$

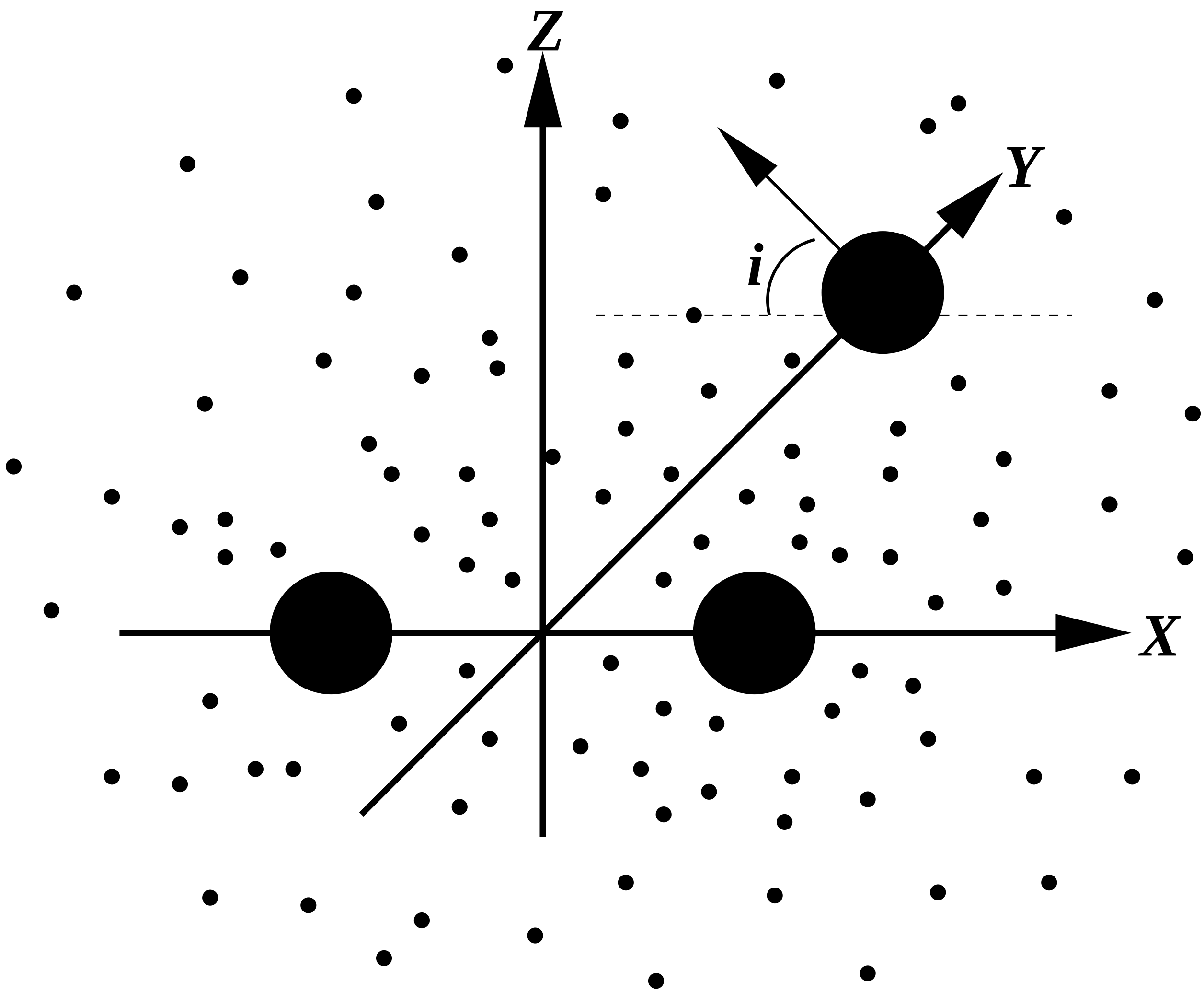

Fig. 1.— Initial setup of BH particles in the parent galaxy is shown schematically. Small dots and large filled circles denote field particles and BH particles, respectively. The arrow shows the initial velocity of the 3rd blackhole. i is the inclination.

$$\mathbf{F}_{fB,i} = \sum_j^{N_B} \frac{-m_{B,j}\mathbf{r}_{fB,ij}}{\left(r_{fB,ij}^2 + \epsilon_{fB}^2\right)^{3/2}}, \tag{5}$$

$$\mathbf{r}_{ff,ij} = \mathbf{r}_{f,i} - \mathbf{r}_{f,j}, \tag{6}$$

$$\mathbf{r}_{fB,ij} = \mathbf{r}_{f,i} - \mathbf{r}_{B,j}. \tag{7}$$

Here $m_{f,i}$ and $\mathbf{r}_{f,i}$ are the mass and the position of field star with index i, and $m_{B,i}$, $\mathbf{r}_{B,i}$ are those of black hole particle i. We used softening parameters ϵ_{ff} and ϵ_{fB} for field star-field star interactions and field star-black hole interactions. Their values are $\epsilon_{ff} = 0.01$ and $\epsilon_{fB} = 10^{-6}$, respectively.

The equation of motion for BH particles is

$$\frac{d^2\mathbf{r}_{B,i}}{dt^2} = \mathbf{F}_{Bf,i} + \mathbf{F}_{BB,i}, \tag{8}$$

where $\mathbf{F}_{Bf,i}$ and $\mathbf{F}_{BB,i}$ are acceleration due to field stars and BH particles, respectively. They are given by

$$\mathbf{F}_{Bf,i} = \sum_j^{N_f} \frac{-m_{f,j}\mathbf{r}_{Bf,ij}}{\left(r_{Bf,ij}^2 + \epsilon_{fB}^2\right)^{3/2}}, \tag{9}$$

$$\mathbf{F}_{BB,i} = \sum_j^{N_B} \frac{-m_{B,j}\mathbf{r}_{BB,ij}}{\left(r_{BB,ij}^2 + \epsilon_{BB}^2\right)^{3/2}} + \sum_j^{N_B} \mathbf{F}_{GW,ij}. \tag{10}$$

We set the softening between BH particles as $\epsilon_{BB} = 0$. The second term, $\mathbf{F}_{GW}$, is the quadrupole approximation for the orbital change due to GW radiation (Damour 1987), which is expressed as

$$\begin{aligned} \mathbf{F}_{GW,ij} \;\; = \;\; & \frac{4G^2 m_{B,i} m_{B,j}}{5r^3c^5}\left[\left(-v^2 + \frac{2Gm_{B,i}}{r} - \frac{8Gm_{B,j}}{r}\right)\mathbf{v},\right. \\ & \left. + \frac{\mathbf{r}\cdot\mathbf{v}}{r^2}\left(3v^2 - \frac{6Gm_{B,i}}{r} + \frac{52Gm_{B,j}}{3r}\right)\mathbf{r}\right], \end{aligned} \tag{11}$$

$$\mathbf{r} = \mathbf{r}_{BB,ij} = \mathbf{r}_{B,i} - \mathbf{r}_{B,j}, \tag{12}$$

$$\mathbf{v} = \mathbf{v}_{BB,ij} = \mathbf{v}_{B,i} - \mathbf{v}_{B,j}, \tag{13}$$

where $\mathbf{v}_{B,i}$ is the velocity of black hole i.

2.3. Numerical Method

We integrated the equations of motions using the 4th order Hermite scheme (Makino & Aarseth 1992) with individual variable time step. In order to apply the Hermite scheme, we need the time derivative of the acceleration. For the gravitational wave term (Eq. 11), we used the following form as the time derivative:

$$\begin{aligned}\frac{d\mathbf{F}_{GW,ij}}{dt} &= \frac{4G^2 m_{B,i} m_{B,j}}{5r^3c^5}\Bigg[\left(-2\left(\mathbf{v}\cdot\mathbf{a}\right)+\frac{3\left(\mathbf{r}\cdot\mathbf{v}\right)v^2}{r^2}\right)\mathbf{v}-v^2\mathbf{a} \\ &+\frac{G}{r}(2m_{B,i}-8m_{B,j})\left(\mathbf{a}-\frac{4\left(\mathbf{r}\cdot\mathbf{v}\right)}{r^2}\mathbf{v}\right) \\ &+\frac{3}{r^2}\left(\left((v^2+\mathbf{r}\cdot\mathbf{a})v^2+2(\mathbf{v}\cdot\mathbf{a})\left(\mathbf{r}\cdot\mathbf{v}\right)-\frac{5\left(\mathbf{r}\cdot\mathbf{v}\right)^2 v^2}{r^2}\right)\mathbf{r}+v^2\left(\mathbf{r}\cdot\mathbf{v}\right)\mathbf{v}\right) \\ &+\frac{1}{r^3}\left(\frac{52}{3}Gm_{B,j}-6Gm_{B,i}\right) \\ &\times\left(\left(v^2+\left(\mathbf{r}\cdot\mathbf{a}\right)\right)\mathbf{r}+\left(\mathbf{r}\cdot\mathbf{v}\right)\mathbf{v}-\frac{6\left(\mathbf{r}\cdot\mathbf{v}\right)^2}{r^2}\mathbf{r}\right)\Bigg],\end{aligned} \tag{14}$$

$$\mathbf{a}=\mathbf{a}_{BB,ij}=\mathbf{a}_{B,i}-\mathbf{a}_{B,j}, \tag{15}$$

where $\mathbf{a_{B,i}}$ is ith black hole acceleration.

In order to calculate the acceleration due to field particles, we used GRAPE-6(Makino et al. 2003), the special purpose computer for the gravitational N-body problem. Forces from BH particles, both Newtonian and gravitational wave terms, are calculated on the host computer.

In all runs, the energy (corrected for the loss of energy through GW radiation)is conserved to 0.1%.

2.4. Correspondent Physical Scales

We interpret the mass unit in our simulation as $10^{10}M_\odot$. Thus, the mass of BH particles is $10^8 M_\odot$. The total mass of $10^{10}M_\odot$ corresponds to the mass of the nucleus of a galaxy containing a BH. In other words, the galaxy model we use is not really the model of the entire galaxy, but only that of the central region. The reason we limit our model to the central region is simply to reduce the number of particles N, in other words, to reduce the calculation cost. This modification means that the potential depth of our model galaxy is somewhat less than that of a real galaxy with the same velocity dispersion. To see the effect of this difference, we have changed the velocity dispersion of the galaxy and investigated

the effect. In the standard runs, we set the velocity dispersion of the initial King model as 300 km/s. In other words, we set the light velocity c in the N-body unit to 706. We tried two different values of velocity dispersion, 600 km/s and 150 km/s, or the value of the light velocity 353 and 1412, respectively. Note that by changing the value of c actually we change the strength of the GW term in equation (11) and thus change the evolution of the BH particles.

3. Result I — Simplest configuration

In this section, we describe the results of runs in which the third BH is initially at rest, to illustrate the typical behavior and the dependence of the results on the artificial parameters of numerical experiments such as the softening and the number of particles. We discuss the results of more general configurations in section 4.

Figure 2 shows the evolution of the semi-major axis a and the eccentricity e of the BH binary in one run of model 64kW7 (hereafter called run 64kW7A). In this and all other figures, the semi-major axis and eccentricity are defined as those of the most strongly bound pair of BH particles. Initially (for time $T < 1.8$), a shows smooth decrease. This is due to the dynamical friction from field stars. The jumps in a and e at $T \sim 1.8$ is the result of first strong encounter between the binary and the third body. In this case, the binary becomes wider and more eccentric. The three BH particles experienced quite complex series of interactions until $T \sim 10$. At this point, the third BH was ejected out, and it took some time before it came back and interacted strongly with the binary at $T \sim 13$. After this interaction, the eccentricity of the binary reached 0.991, resulting in the quick orbital decay by GW emission.

In this case, the high eccentricity is realized only after the last strong interaction. However, there are several other periods when the eccentricity was large, like around $T = 7$ ($e \sim 0.97$) and $T = 9$ ($e \sim 0.94$). High values of eccentricity are not very unusual after strong interaction events.

In figure 3, orbits of three blackholes in run 64kW7A projected on the $x - y$ plane are shown. The top panel shows the trajectory of two BH particles which are initially in a binary, for the period of $0 \le T \le 1.8$. The third BH particle is out of the plotting region and thus not shown. Initially, the orbits decay quickly. As a result the orbital period becomes shorter and the change in one orbit becomes smaller. In the meantime, the center of mass of the binary moves along the y axis, because of the gravitational force from the third BH particle which was initially located at (0,1,0) and falling down to the center of the galaxy.

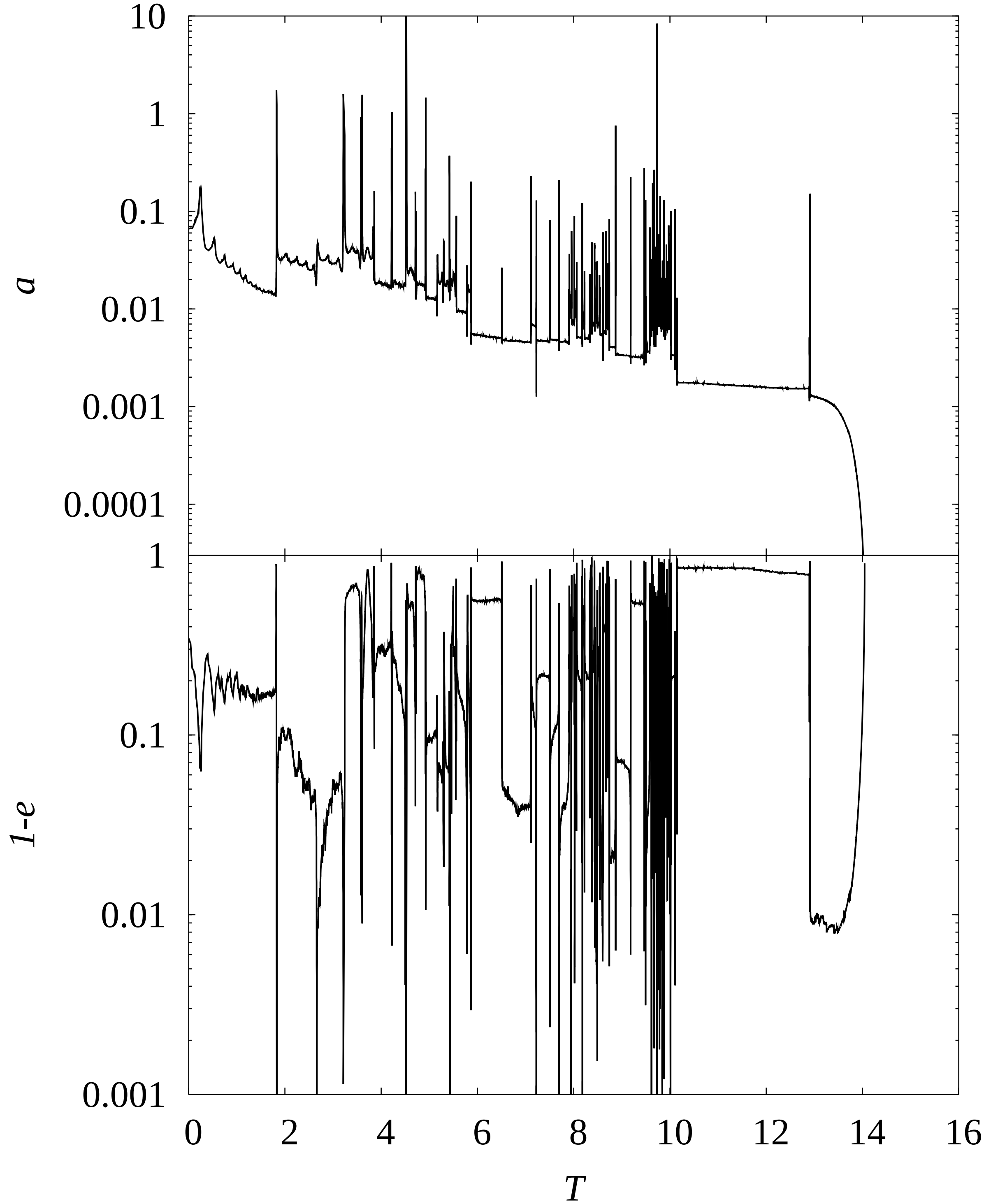

Fig. 2.— Evolution of semi-major axis a (top panel) and eccentricity e (bottom panel, $1-e$ is shown) for run 64kW7A.

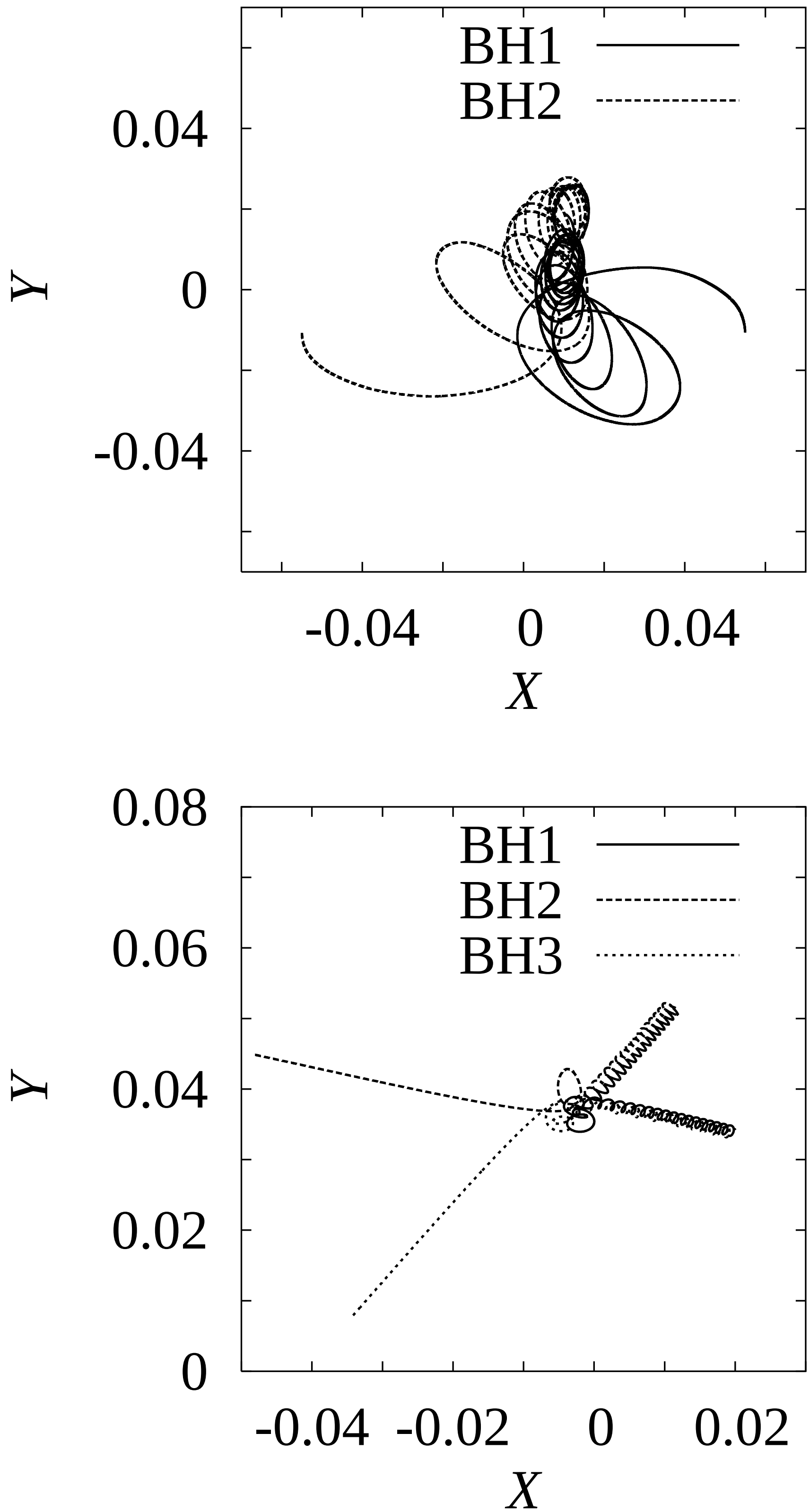

Fig. 3.— Trajectories of the BHs before three body encounter($T = 0 - 1.8$) and just before and after the last strong encounter ($T = 12.85 - 12.95$).

The bottom panel of figure 3 shows the last strong encounter. The binary BH (of BHs numbered 1 and 3) comes in from the right side, and the third BH (BH2) comes in from the left side. After the complex encounter, a new binary of BHs 1 and 2, with the very high eccentricity, flies out to upper-right corner and BH3 to the lower-left corner.

3.1. Merging Probability

The last column of table 2 shows the fraction of runs in which merger occurred. The divisor is the total number of runs performed for the model and dividend is the number of runs in which merging occured. For runs of model 64kW7 with velocity dispersion $\sigma = 300$km/s, in three out of five runs the BH binary merged before single BH is kicked out of the parent galaxy. In two other cases, a single BH was kicked out. For runs of model 64kW7High ($\sigma = 600$km/s), two out of three resulted in merging. All of runs of model 64kW7Low ($\sigma = 150$km/s) resulted in ejection.

The total potential depth of typical giant ellipticals is somewhere between galaxy models 64kW7 and 64kW7High. Thus, we can conclude that a triple BH system, if formed in giant ellipticals, is likely to end up in merging of two of the three BHs, leaving out a binary BH system. THe Ejection of one BH is not impossible, though. In the remaining part of this section, we check the effect of artificial parameters such as the softening and the number of particles.

3.2. Effect of the softening

In figure 4, we show the evolution of semi-major axis of binaries of two runs with different softening parameters between FS-FS. Solid and dashed curves correspond to runs with $\epsilon_{ff} = 0.01$ and 0.001, respectively. Though the details are different, the overall evolution is similar. We performed five runs with $\epsilon_{ff} = 0.001$, and one runs ended up in merging. This result is consistent with that for $\epsilon_{ff} = 0.01$.

3.3. Effect of the number of particles

In figure 5, we show the evolution of semi-major axis of binaries in runs with different number of particles for the parent galaxy. Again, the overall behavior is not much different. For $N = 16$k and $N = 128$k, we again performed ten and five runs, and four runs each ended up in merging. We can conclude N has little effect on the result.

Table 1. Model Parameters

Parameter	Symbol	Value
Mass of galaxy	M_{gal}	1.0
Mass of FS	m_f	$1/16384 - 1/131072$
Mass of BH	m_B	0.01
Number of FS	N_f	$16 - 128k$
Number of BH	N_B	3
Gravitational constant	G	1
Total energy	E_{gal}	-0.25
Softening between BH and BH	ϵ_{BB}	0.0
Softening between BH and FS	ϵ_{Bf}	10^{-6}
Softening between FS and FS	ϵ_{ff}	$0.01 - 0.001$

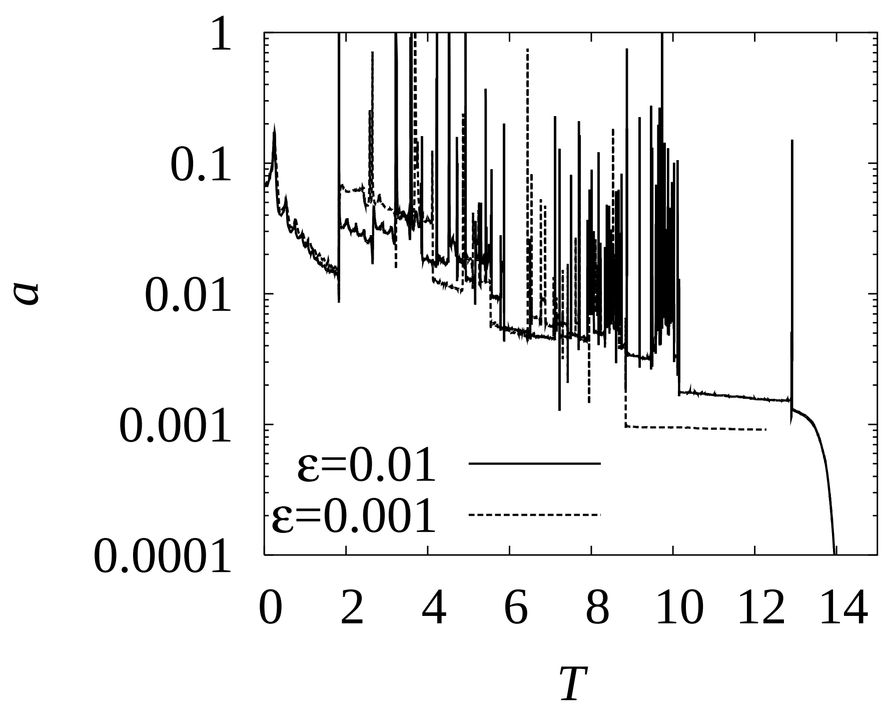

Fig. 4.— The evolution of semi-major axis for runs with different values of the softening parameter ϵ_{ff} for field particles. Solid and dashed curves are the results of runs with $\epsilon_{ff} =$ 0.01 and 0.001, respectively. All other parameters are the same as those in model 64kW7.

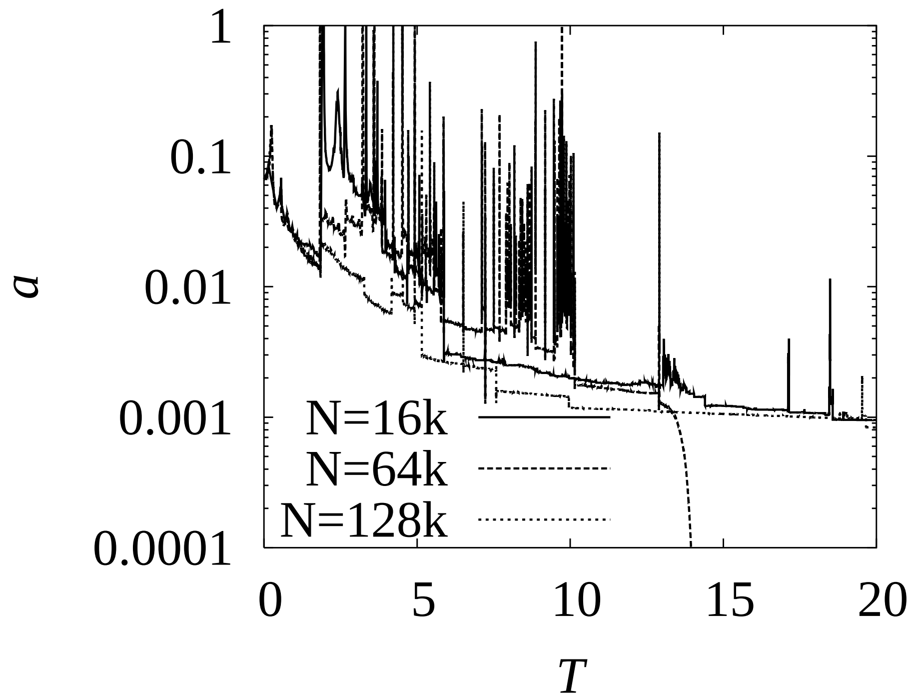

Fig. 5.— The evolution of semi-major axis for different values of N. Solid, long-dashed and short-dashed curves are the results of runs with N = 16k, 64k and 128k, respectively. All other parameters are the same as those in model 64kW7.

This result is quite different from that for the evolution of two massive BHs (one BH binary) in the central region of the galaxy, where the number of particles, *i.e.*, the relaxation timescale, determines the evolution timescale of the BH binary. In the case of two BHs, the loss cone is soon depleted, and the only way for the field particles to interact with the BH binary is to enter the loss cone through two-body relaxation. Thus, it is quite natural that the evolution timescale depends on N. However, in the case of a triple BH system, three BHs alone determines the evolution, and field particles works just as the background potential and the source for the dynamical friction. Thus, there is nothing like loss-cone depletion.

3.4. Effect of the initial galaxy model

In addition to models with $W_0 = 7$, we tried three runs with $W_0 = 11$. As can be seen in table 2, there is not much difference in the final outcome. This result looks a bit counter-intuitive, since, as we have seen in section 3.1, the merging probability does depend on the depth of the potential relative to the speed of the light.

The reason why the outcome is not much different in cases with $W_0 = 7$ and 11 is that initial binary BH hardens much more quickly in models with $W_0 = 11$ than in models with $W_0 = 7$. As a result, ejection of BH occurs after smaller number of triple interactions and the chance to reach high eccentricity is smaller for higher central density. This difference more than compensates the deeper potential.

4. Result II — General configurations

In section 3, we considered the simplest case in which a binary is at the center of the galaxy and the third BH particle falls from around the half-mass radius. This is clearly a

Table 2. Models for section 3

Model name	N_f	W_0	ϵ_{ff}	σ	c	merging probability
16kW7	16k	7	0.01	300km/s	706	4/10
32kW7	32k	7	0.01	300km/s	706	2/5
64kW7	64k	7	0.01	300km/s	706	3/5
128kW7	128k	7	0.01	300km/s	706	4/5
64kW7High	64k	7	0.01	600km/s	353	2/3
64kW7Low	64k	7	0.01	150km/s	1412	0/3
64kW11	64k	11	0.001	300km/s	677	1/3
64kW7Eps	64k	7	0.001	300km/s	706	1/5

rather special case, and the result may be biased. In this section we consider several different configurations.

Figure 6 shows the result of runs in which the third BH has the initial velocity perpendicular to the orbital plane of the binary BH (runs 64kI90Vx, see Table 3). The case with $v_z = 0.1$ is qualitatively similar to the result of free-fall runs (figure 2). Orbital elements of the binary shows noncontinuous changes after strong encounters with the third BH. The time interval for the encounter seems to be longer, due to the larger total angular momentum of the triple BH system.

The case with $v_z = 0.3$ shows quite different behavior. The eccentricity shows periodic-like change for $T = 11 \sim 18$, but the semi-major axis does not show much change during the same period. This behavior is driven by the Kozai mechanism. The period and magnitude of the oscillation change in time, because the orbital elements of both the inner and outer binaries changes due to the interaction with other field stars. Around $T = 18$, the hierarchical system of three BHs became unstable, and strong three-body interactions followed.

In the case of $v_z = 0.7$, the inner binary merged in the first cycle of the Kozai cycles.

Figure 7 shows the results of three runs with different initial inclination of the orbital plane of the third BH. As we've seen in figure 6, in the extreme case of large v_z perpendicular to the binary orbital plane, the end result is the merging in the first Kozai cycle. In the case of $i = \pi/3$, Kozai cycle drives oscillations of large amplitude, and semi-major axis shrinks each time when the eccentricity reached the peak. In the case of $i = \pi/6$, BHs neither merge nor escape until $T = 200$. Kozai cycles are also observed. The maximum eccentricity in one cycle is, however, not high enough to drive the merging of inner binary. In the case of the coplanar orbits, Kozai cycle of a small magnitude is observed for $T = 20$ to 90. Since the initial orbit is coplanar, there cannot be any Kozai cycle if three BHs remain exactly in the same orbital plane. However, since the number of particles is finite, fluctuation in the forces from field particles introduces random changes in the inclination of inner and outer binaries, resulting in the configuration for which the Kozai cycle can show up.

To summarize the results of 3D triple cases, a hierarchical triple BH system is often formed when the third BH falls in. In such a triple system, a stable oscillation in eccentricity (and inclination) driven by Kozai mechanism is often observed. In a fair fraction of the cases, this oscillation results in the merging of the inner binary by GW emission.

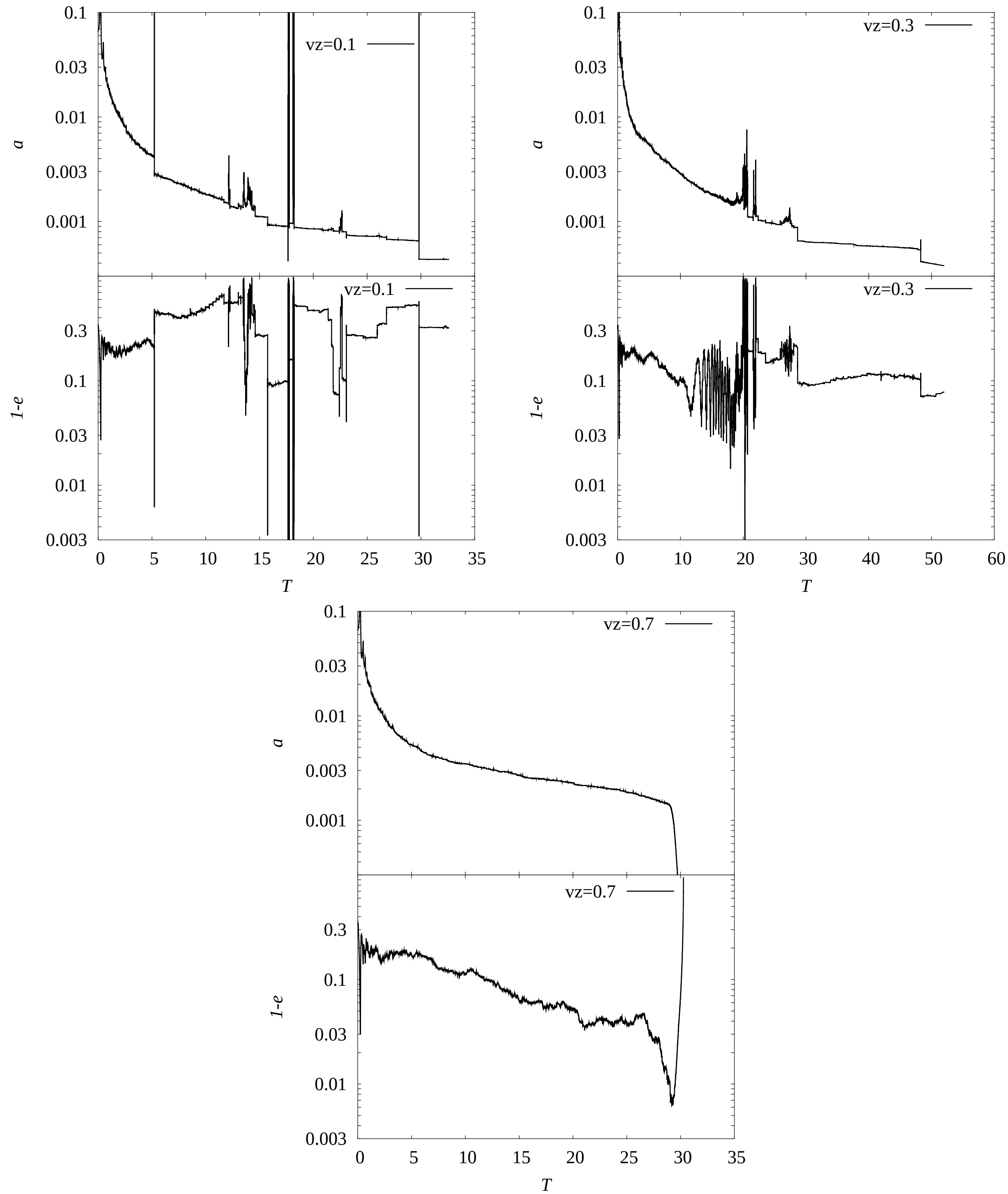

Fig. 6.— The evolution of the semi-major axis a and eccentricity e for runs with non-zero initial V_z of the third BH particle. Left top, right top, and bottom panels show the results for $V_z = 0.1, 0.3, 0.7$, respectively.

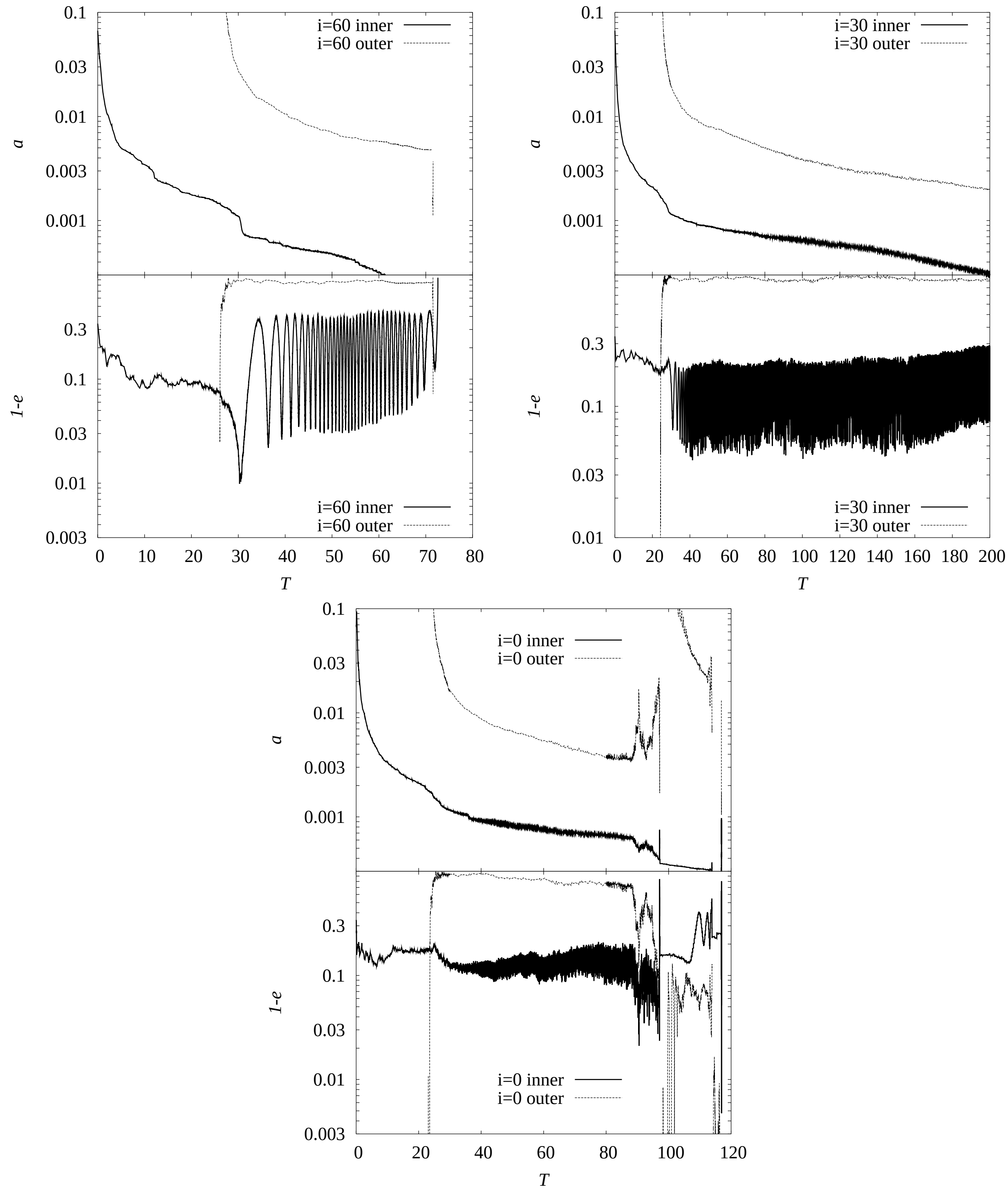

Fig. 7.— The evolution of the semi-major axis a and eccentricity e for runs with initial $|V| = 0.7$ for the third BH particle. Initial velocity of the third BH is in yz plane and the angle with the xy plane is 60 (left top), 30 (right top) and 0 (bottom) degree. Each panel shows the semi-major axis and the eccentricity of inner (thick solid line) and outer binaries (thin dashed line).

5. Discussion

5.1. Escaping cases and free-floating BHs

We have seen that the merging of the BH binary is more likely than the ejection of single BH (or also the binary BH). However, the ejection is not a rare event. Out of the five runs with $N = 64k$ and $\sigma = 300$km/s listed in table 2, two runs ended up in the ejection. Though this result is somewhat biased by the shallow galactic potential, runs with deeper potential still gives a fair number of escaping events.

Thus, our result suggests that, though in most cases the binary BH do merge, there may be cases in which one or more BHs are kicked out. In other words, there may be a fair number of massive BHs free-floating in intergalactic field.

An interesting question is if such a floating BH can be observed. One possibility is through strong lensing, but since the Einstein radius, when placed around $z = 0.5$, is around 10^{-2} arcsec, it would be hard to detect it even with HST. Another possibility is the detection of X-ray emission by Bondi accretion of intracluster hot gas, if BH is in a cluster of galaxy. The luminosity L is given by

$$L = \eta \dot{m}_B c^2, \tag{16}$$

where η is the radiative efficiency, $\dot{m}_B$ is the accretion rate and c is the light velocity. The accretion rate is given by

$$\dot{m}_B = F(\gamma)\pi G^2 m_B^2 \rho_\infty / a_\infty^3, \tag{17}$$

where $F(\gamma)$ is a function of adiabatic index of the gas γ, ρ_∞ and a_∞ are density and sound speed at infinity, respectively (Bondi 1952). We take $\eta = 0.1$ and $F(\gamma) = 1$. For a BH of $10^8 M_\odot$ in the Virgo cluster, whose typical hot gas density and sound speed is given by

$$\rho_\infty \sim 1.7 \times 10^{-28} \frac{n}{1.0 \times 10^{-4}\mathrm{cm}^{-3}} \mathrm{g/cm}^3, \tag{18}$$

$$a_\infty \sim 4.3 \times 10^7 \left(\frac{kT}{2\mathrm{keV}}\right)^{1/2} \mathrm{cm/s}, \tag{19}$$

Table 3. Models for section 4

Model name	N_f	W_0	i	$\|V\|$	σ	c	final state
64kI90V1	64k	7	90	0.1	300km/s	706	escape
64kI90V3	64k	7	90	0.3	300km/s	706	escape
64kI90V7	64k	7	90	0.7	300km/s	703	merge
64kI60V7	64k	7	60	0.7	300km/s	703	merge
64kI30V7	64k	7	30	0.7	300km/s	703	neither merge nor escape by $T = 200$
64kI0V7	64k	7	0	0.7	300km/s	703	merge

where n and kT are the number density and temperature of the intracluster hot gas and we adopted $n = 1.0 \times 10^{-4}\mathrm{cm}^{-3}$ and $kT = 2\mathrm{keV}$ (Kikuchi et al. 2000) as typical values, thus the luminosity would be

$$L_{virgo} = 9.0 \times 10^{19} \dot{m}_B \mathrm{erg/s} \tag{20}$$

$$= 7.9 \times 10^{37} \left(\frac{m_B}{10^8 M_\odot}\right)^2 \left(\frac{n}{1.0 \times 10^{-4}\mathrm{cm}^{-3}}\right) \left(\frac{kT}{2\mathrm{keV}}\right)^{-3/2}. \tag{21}$$

Thus, it can be observed as a moderately luminosity X-ray source. The question is how it can be discriminated from AGNs.

5.2. Conclusion

In this paper, we investigated the dynamical evolution of triple black hole systems in the central region of a galaxy. We found that in most of cases two black holes merge through GW radiation. The merging timescale by GW radiation is greatly reduced by high eccentricity of the binary BH. The high eccentricity is realized by two different mechanisms. First one is the random change of the eccentricity after binary-single BH interaction as suggested by Makino & Ebisuzaki (1994), and the other is the Kozai mechanism(Kozai 1962). Even so, there may be a fair number of free-floating BHs ejected out of the parent galaxy by a slingshot. They might be observed as X-ray sources, if they stay within a cluster of galaxies.

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